Math 22
Quiz 4
Wed Oct 24, 2012

NAME YOU ASKED TO BE CALLED IN CLASS: ____________________

SCORE: _____ / 35 POINTS

1. You may use the result of exercise 26 in section 4.4 without proving it.
2. You may NOT use the results of example 4.2.3 in section 4.2 unless you write formal proofs of them.
3. You may use the property that all integers are either even or odd, but NOT the property that consecutive integers have opposite parity.

Find the values of (-73) div 8 and (-73) mod 8. **Justify your answers very briefly.** **You do NOT need to write a formal proof.**

SCORE: _____ / 4 PTS

2 $-73 = 8 \times (-10) + 7$

(-73) div $8 = -10$ and (-73) mod $8 = 7$

1 1

One of the following statements is true and one is false.
Write a formal proof for the true statement, and show that the false statement is false.

SCORE: _____ / 12 PTS

[a] For all integers a and n, if $a \mid n^2$ and $a \le n$, then $a \mid n$.

[b] For all integers a and n, $a \mid n$ is necessary for $a^2 \mid n$.

[a] is false.

Let $a = 4$ and $n = 6$.
$4 \mid 36$ and $4 \le 6$, but $4 \nmid 6$.

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[b] is true.

The statement can be reworded as "For all integers a and n, if $a^2 \mid n$, then $a \mid n$".

PROOF:

1 1

Let a and n be particular but arbitrary integers such that $a^2 \mid n$.
So, 1 $n = ka^2 = (ka)a$ 1 by def'n of $|$, 1
1 where $ka \in Z$ by closure of Z under $\times$. 1
So, $a \mid n$ by def'n of $|$.
1

Write the formal definition of "prime". **Use proper English and mathematical notation.**

SCORE: _____ / 4 PTS

An integer n is prime if and only if
$n > 1$ and
for all positive integers r and s,
if $n = rs$ then $r = 1$ or $s = 1$.

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Write a formal proof for the statement

SCORE: _____ / 15 PTS

"For all integers n, $n^2 - n - 3$ is odd"

PROOF:

1 Let n be a particular but arbitrary integer.
2 $n = 2q$ or $n = 2q + 1$ by QRT. 1

1 CASE 1 ($n = 2q$):
$n^2 - n - 3 \;=\; 4q^2 - 2q - 3 \;=\; 2(2q^2 - q - 2) + 1$ 1
where 1 $2q^2 - q - 2 \in Z$ by closure of Z under $\times$ and $-$. 1
So, $n^2 - n - 3$ is odd by def'n of odd. 1

1 CASE 2 ($n = 2q + 1$):
$n^2 - n - 3 \;=\; 4q^2 + 4q + 1 - 2q - 1 - 3 \;=\; 4q^2 + 2q - 3 \;=\; 2(2q^2 + q - 2) + 1$ 1
where 1 $2q^2 + q - 2 \in Z$ by closure of Z under $\times$ and $-$. 1
So, $n^2 - n - 3$ is odd by def'n of odd. 1

So, for all integers n, $n^2 - n - 3$ is odd. 1
